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How does a startup respond to acquirers? A real options analysis

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1. Introduction

Recently, most industries have increased concentration levels, and the difference between profitability of giant firms and others has greatly increased. In such circumstances, many startups tend to be acquired by large firms rather than growing by initial public offerings and investments as independent firms (e.g., [2]). This paper investigates such a firm's optimal response to acquirers arriving sequentially.

2. Model Setup

Consider a startup that is generating continuous streams of cash flows $X(t)$, which follows

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t)\ (t > 0), \quad X(0) = x,$$

where $B(t)$ denotes the standard Brownian motion, and $\mu, \sigma(> 0)$ and $x(> 0)$ are constants. The net present value of cash flows becomes $\pi(x) = x/(r - \mu)$, where r denotes the discount rate. Consider high- and low-type acquirers, who value $a_H\pi(X(t)) - I_a$ and $a_L\pi(X(t)) - I_a$, respectively for the firm, where $\Delta a = a_H - a_L > 0, a_L > 1$, and sunk cost $I_a > 0$. High and low types approach the startup, following the Poisson processes with arrival rates λ_H and λ_L, respectively. The Poisson processes and $X(t)$ are independent. The startup also pays sunk cost $I_b(> 0)$ when acquired, and the total costs become $I = I_a + I_b$. The startup chooses between accepting or rejecting acquisition to maximize the expected firm value. If the startup rejects an acquirer, the firm has to wait until another acquirer arrives.

Although the model is stylized, it captures irreversibility, illiquidity, unpredictability, and price uncertainty in acquisition. In the model, I explore the startup's sellout timing and pricing decisions affected by these frictions.

3. Model Solutions

First, consider the symmetric information case where the acquirer types are observable to the startup. When an i-type arrives at t, the firm value becomes $\max\{a_i\pi(X(t)) - I, V(X(t))\}$, where $V(X(t))$ denotes the firm value while sellout is infeasible, because the firm optimizes the choice between accepting and rejecting the acquirer. The function $V(x)$ satisfies:

$$\begin{aligned}
&\mu x V'(x) + 0.5\sigma^2 x^2 V''(x) + x = rV(x) \quad (0 < x < x_H^*),\\
&\mu x V'(x) + 0.5\sigma^2 x^2 V''(x) + \lambda_H(a_H\pi(x) - I - V(x))\\
&\ + x = rV(x) \quad (x_H^* < x < x_L^*),\\
&\mu x V'(x) + 0.5\sigma^2 x^2 V''(x) + \bar{\lambda}(\bar{a}\pi(x) - I - V(x))\\
&\ + x = rV(x) \quad (x > x_L^*),
\end{aligned}$$

where, $[x_H^*, \infty)$ and $[x_L^*, \infty)$ represent the acceptance regions for high and low types, respectively, and I define $\bar{\lambda} = \lambda_H + \lambda_L$ and $\bar{a} = (\lambda_H a_H + \lambda_L a_L)/\bar{\lambda}$. Note that x_L^* can be infinity. The function $V(x)$ is continuously differentiable at x_i^* and satisfies boundary conditions

$$\begin{aligned}
&V(x_i^*) = a_i\pi(x_i^*) - I \quad (i = H, L),\\
&\lim_{x\to 0} V(x) = 0\\
&\lim_{x\to\infty} V(x)/x < \infty,
\end{aligned}$$

where the first condition means that the firm is indifferent to whether to accept or reject an i-type proposal at the threshold x_i (i.e., the optimality of x_i).

I define the notations

$$\begin{aligned}
\beta_y &= 0.5 - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - 0.5)^2 + 2(r+y)/\sigma^2},\\
\gamma_y &= 0.5 - \mu/\sigma^2 - \sqrt{(\mu/\sigma^2 - 0.5)^2 + 2(r+y)/\sigma^2},
\end{aligned}$$

for an arbitrary parameter $y > 0$, and I simplify the notations by $\beta = \beta_0$ and $\gamma = \gamma_0$ for $y = 0$. By solving the differential equations with the boundary conditions, I have the following proposition:

Proposition 1 *If condition*

$$a_L - 1 \le \frac{\lambda_H(a_H - 1)}{r + \lambda_H - \mu} \tag{1}$$

holds, the firm takes the high price strategy, i.e., $x_L^* = \infty$. The firm value and sellout threshold agree with those in the model with only high types. If condition (1) does not hold, the firm takes the flexible strategy, i.e., $x_L^* < \infty$. The firm value $V(x)$ is given by

$$V(x) = \begin{cases} \pi(x) + B_1 x^{\beta} \quad (x < x_H^*), \\ \pi(x) + \dfrac{\lambda_H(a_H - 1)\pi(x)}{r + \lambda_H - \mu} - \dfrac{\lambda_H I}{r + \lambda_H} \\ +B_2 x^{\beta_{\lambda_H}} + B_3 x^{\gamma_{\lambda_H}} \quad (x \in [x_H^*, x_L^*)), \\ \pi(x) + \dfrac{\bar{\lambda}(\bar{a} - 1)\pi(x)}{r + \bar{\lambda} - \mu} - \dfrac{\bar{\lambda} I}{r + \bar{\lambda}} + B_4 x^{\gamma_{\bar{\lambda}}} \\ (x \geq x_L^*). \end{cases}$$

The coefficients B_i $(i = 1, 2, 3, 4)$ are expressed by the sellout thresholds x_i^* $(i = H, L)$, which are the solutions to the two nonlinear equations (the details are omitted).

This proposition shows the two types of solutions, i.e., the high price strategy, where the firm accepts only a high type regardless of market environments, and the flexible strategy, where the firm accepts even a low type in good environments. The solution of the high price strategy case is essentially equal to that of an optimal stopping problem constrained within Poisson jump times in [1]. [3] also solves this type of model to explore the illiquidity of sellout opportunities on bankruptcy. On the other hand, the solution in the flexible strategy case is newly obtained.

Next, consider the asymmetric information case where the acquirer types are unobservable to the startup. In the acceptance region for low types, the startup sells out at a low price to any type. In other words, a high type collects the information rents. As in the symmetric information case, I have the following proposition:

Proposition 2 *If condition*

$$a_L - 1 - \frac{\Delta a \lambda_H}{\lambda_L} \leq \frac{\lambda_H(a_H - 1)}{r + \lambda_H - \mu} \tag{2}$$

holds, the firm takes the high price strategy, i.e., $x_L^{**} = \infty$. The firm value and sellout threshold agree with those in the model with only high types. If condition (2) does not hold, the firm takes the flexible strategy, i.e., $x_L^{**} < \infty$. The firm value $U(x)$ is given by

$$U(x) = \begin{cases} \pi(x) + \tilde{B}_1 x^{\beta} \quad (x < x_H^{**}), \\ \pi(x) + \dfrac{\lambda_H(a_H - 1)\pi(x)}{r + \lambda_H - \mu} - \dfrac{\lambda_H I}{r + \lambda_H} \\ +\tilde{B}_2 x^{\beta_{\lambda_H}} + \tilde{B}_3 x^{\gamma_{\lambda_H}} \quad (x \in [x_H^{**}, x_L^{**})), \\ \pi(x) + \dfrac{\bar{\lambda}(a_L - 1)\pi(x)}{r + \bar{\lambda} - \mu} - \dfrac{\bar{\lambda} I}{r + \bar{\lambda}} + \tilde{B}_4 x^{\gamma_{\bar{\lambda}}} \\ (x \geq x_L^{**}). \end{cases}$$

The coefficients $\tilde{B}_i$ $(i = 1, 2, 3, 4)$ are expressed by the sellout thresholds x_i^* $(i = H, L)$, which are the solutions to the two nonlinear equations (the details are omitted).

The opportunity cost of selling the firm at a low price is higher under asymmetric information because it includes the loss from selling the firm at a low price to a high type (i.e., the information rents to a high type). Then, the left-hand side of (2) has the extra term $-\Delta a \lambda_H / \lambda_L$, and condition (2) is looser than condition (1). The notable region $a_L - 1 - \Delta a \lambda_H / \lambda_L \leq \lambda_H(a_H - 1)/(r + \lambda_H - \mu) < a_L - 1$ arises, where the firm takes the high price strategy only in the asymmetric information case.

I explore the firm value, sellout timing, high-price sellout probability, and social loss due to asymmetric information in numerical examples. In the presentation, I report the results in detail.

References

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[3] M. Nishihara and T. Shibata. The effects of asset liquidity on dynamic sell-out and bankruptcy decisions. *European Journal of Operational Research*, 288:1017–1035, 2021.