

## CORDON AND AREA ROAD PRICING IN RADIAL-ARC NETWORK

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*Abstract* This paper proposes a continuous network model for determining the size of the toll area and toll level in cordon and area road pricing. Cordon pricing charges a toll to vehicles passing a cordon line surrounding a designated area, whereas area pricing charges a toll to all vehicles driving inside the area. Analytical expressions for the traffic volume and toll revenue are obtained for a circular city with a radial-arc network. The analytical expressions demonstrate how the size of the toll area and toll level affect the traffic volume and toll revenue. Comparing cordon and area pricing shows that area pricing is superior to cordon pricing in both reducing traffic volume in the toll area and generating revenue.

**Keywords:** Transportation, toll area, toll level, traffic volume, toll revenue, continuous network model

### 1. Introduction

Road pricing has attracted attention as a means to reduce traffic congestion. Road pricing encourages travelers to adjust all aspects of their behavior such as number of trips, route, and mode of transport, and thus has a big advantage over other travel demand management policies [4].

The most popular road pricing system is cordon pricing in which vehicles passing a cordon line surrounding a designated area are charged a fixed toll. The cordon pricing has been adopted in Singapore, Stockholm, and Milan. May and Milne [13] compared cordon-based, distance-based, time-based, and delay-based pricing systems. Akiyama et al. [1] compared cordon pricing with congestion pricing on existing toll roads. Sumalee [19] presented a branch-tree framework for finding the optimal cordon location and toll level. A large number of solution methods for the problem have been proposed [5, 18, 25]. Extensions of the problem include multi-layered and multi-centered cordon [26] and time-dependent pricing [7, 27].

Not only cordon pricing but also area pricing has been studied and implemented. In the area pricing, all vehicles driving inside a designated area are charged a fixed toll. Implementing the area pricing is thus more difficult than the cordon pricing. The area pricing has been adopted in London. Maruyama and Harata [11] and Maruyama and Sumalee [12] compared the performance of the cordon and area pricing using a trip-chain equilibrium model. Takaki et al. [20] and Takaki et al. [21] developed an algorithm for determining the optimal shape of toll area and toll level. Zheng et al. [28] considered time-dependent area pricing.

Continuous network models have also been used to analyze road pricing. The discrete network models reviewed above aim to develop efficient algorithms applicable to actual road networks, whereas the continuous network models aim to examine fundamental relationships between variables. The continuous models often yield analytical solutions that are easy

to interpret and comprehend, thus supplementing the discrete models. Mun et al. [15] obtained the optimal cordon location and toll level using an urban spatial model of a linear monocentric city. The model was extended by Mun et al. [16] to a non-monocentric city, Verhoef [24] to incorporate land and labor markets, Li et al. [9] to consider the interaction between auto and bus, and Tsai and Lu [22] to multiple-cordon. Ho et al. [6] applied a continuum traffic equilibrium model to the cordon pricing problem in an arbitrary shaped city. Li et al. [10] examined the effect of air pollution cost on the optimal cordon location and toll level in a circular city. In the literature of continuous models, few studies have considered area pricing.

In this paper, we propose a continuous network model for determining the size of the toll area and toll level in cordon and area road pricing. The model yields analytical expressions for the traffic volume and toll revenue, leading to a clear understanding of the basic effect of road pricing. The model will therefore supply building blocks for designing road pricing systems. Other characteristics of the model are as follows. First, the model uses a radial-arc network, which can be found in many cities such as Tokyo, Paris, and Moscow. Second, the model is based on a non-monocentric city where trips occur between any two points in the city. Finally, the model deals with both cordon and area pricing.

The remainder of this paper is organized as follows. The next section develops a radial-arc network model. The following section examines the effects of the size of the toll area and toll level on the traffic volume and toll revenue. The penultimate section compares the cordon and area pricing. The final section presents concluding remarks.

## 2. Radial-Arc Network Model

Consider a circular city with radius  $a$ , as shown in Figure 1. The city has a dense radial-arc network. A toll area is represented as a circle with radius  $b$  located at the center of the city. Cordon pricing charges a toll to vehicles passing or entering the area, whereas area pricing charges a toll to all vehicles driving inside the area. The objectives of road pricing are to reduce the traffic volume in the toll area and to generate the toll revenue. The former directly reduces the traffic congestion in the area, whereas the latter indirectly reduces the traffic congestion by using the revenue to improve infrastructure.

Let  $t$  be the toll level and  $\alpha$  be the travel cost per unit distance. The travel cost  $C$  for trips of length  $R$  is defined as

$$C = \alpha R + t. \quad (2.1)$$

Every traveler is assumed to use the least cost route. Since the travel cost also depends on the travel time in practice, this assumption can be applied to the case where the travel speed is regarded as constant throughout the city. If no toll is charged, i.e.,  $t = 0$ , the travel distance  $R$  is reduced to the radial-arc distance. The radial-arc distance  $S$  between two points  $(r_1, \theta_1)$  and  $(r_2, \theta_2)$  in the polar coordinate is defined as

$$S = \begin{cases} |r_1 - r_2| + \min\{r_1, r_2\}\varphi, & 0 \leq \varphi < 2, \\ r_1 + r_2, & 2 \leq \varphi \leq \pi, \end{cases} \quad (2.2)$$

where  $\varphi = \min\{|\theta_1 - \theta_2|, 2\pi - |\theta_1 - \theta_2|\}$  [8]. Both radial and arc roads are used if  $\varphi < 2$ , whereas only radial roads are used if  $\varphi \geq 2$ , as shown in Figure 1. The radial-arc distance is a good approximation for the actual travel distance in cities with a radial-arc network [23]. The travel demand  $D$  depends on the travel cost and is expressed as

$$D = D_0 e^{-\beta C}, \quad (2.3)$$

where  $D_0$  is the travel demand when  $C = 0$  and  $\beta (> 0)$  is a parameter for elasticity. The travel demand decreases with the trip length  $R$  and the toll level  $t$ . The exponential function, which is common in spatial interaction models [17], allows us to obtain analytical expressions for the traffic volume and toll revenue.

Assume that origins and destinations are uniformly distributed in the city. The uniform distribution serves as a basis for further analysis with more realistic distributions. In fact, the uniform distribution has frequently been used in continuous transportation models [2, 3, 14]. The traffic in the city is classified into four groups according to the location of origin and destination. If both origin and destination are outside the toll area, the traffic is called *through traffic*. If origin is outside (inside) the toll area and destination is inside (outside) the toll area, it is called *inward (outward) traffic*. If both origin and destination are inside the toll area, it is called *city traffic*.

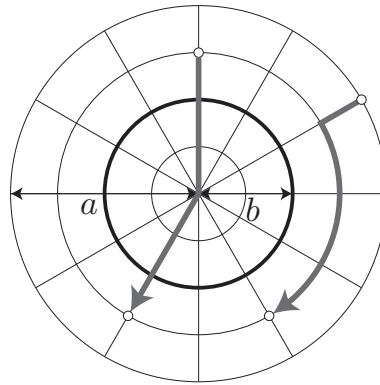


Figure 1: Circular city with a radial-arc network

### 3. Traffic Volume and Toll Revenue

In this section, we examine the effects of the size of the toll area and toll level on the volume of through, inward, outward, and city traffic, and the toll revenue by each traffic.

Let  $(r_1, \theta_1)$  ( $b \leq r_1 \leq a, 0 \leq \theta_1 < 2\pi$ ) and  $(r_2, \theta_2)$  ( $b \leq r_2 \leq a, 0 \leq \theta_2 < 2\pi$ ) be origin and destination of through traffic, respectively. Through traffic passes the toll area if the travel cost of passing the toll area is smaller than that of making a detour around the toll area. The condition of passing the toll area is then

$$2ab + t \leq ab\varphi \Leftrightarrow \varphi \geq \frac{t}{\alpha b} + 2. \tag{3.1}$$

Travel routes for trips of  $\theta_1 = 0$  are shown in Figure 2a. Through traffic passes the toll area if destination is inside the gray region in the figure. The travel cost of through traffic passing the toll area  $C^T$  is given by

$$C^T = \alpha(r_1 + r_2) + t. \tag{3.2}$$

Substituting into (2.3) and integrating with respect to  $r_1, \theta_1, r_2,$  and  $\theta_2$  yield the volume of through traffic passing the toll area

$$\begin{aligned} V^T &= \int_0^{2\pi} \int_b^a \int_{t/(\alpha b)+2}^{\pi} \int_b^a 2D_0 \exp[-\beta\{\alpha(r_1 + r_2) + t\}] r_1 r_2 dr_2 d\theta_2 dr_1 d\theta_1 \\ &= \frac{4\pi D_0}{\alpha^4 \beta^4} \left( \pi - 2 - \frac{t}{\alpha b} \right) \{(\alpha\beta b + 1)e^{\alpha\beta a} - (\alpha\beta a + 1)e^{\alpha\beta b}\}^2 e^{-\beta\{2\alpha(a+b)+t\}}. \end{aligned} \tag{3.3}$$

The volume of through traffic passing the toll area  $V^T$  is shown in Figure 2b, where  $a = 1, D_0 = 1, \alpha = 1, \beta = 1$ . If the toll area is small, much traffic passes the toll area at  $t = 0$  but  $V^T$  decreases rapidly with  $t$ . On the other hand, if the toll area is large,  $V^T$  is small at  $t = 0$  but decreases gradually with  $t$ , and thus a higher toll is required to reduce the volume of through traffic passing the toll area. The toll level that achieves  $V^T = 0$  is

$$t^\dagger = (\pi - 2)\alpha b, \tag{3.4}$$

which is proportional to the radius of the toll area  $b$ . The toll revenue by through traffic  $T^T = tV^T$  is shown in Figure 2c. The toll revenue has a maximum between  $t = 0$  and  $t = t^\dagger$ . The toll level that maximizes the toll revenue by through traffic is

$$t^T = \frac{(\pi - 2)\alpha\beta b + 2 - \sqrt{(\pi - 2)^2\alpha^2\beta^2b^2 + 4}}{2\beta}. \tag{3.5}$$

As the toll area becomes larger, a higher toll is required to maximize the toll revenue. Since both  $t^\dagger$  and  $t^T$  depend on  $b$ , the size of the toll area and toll level should be determined simultaneously.

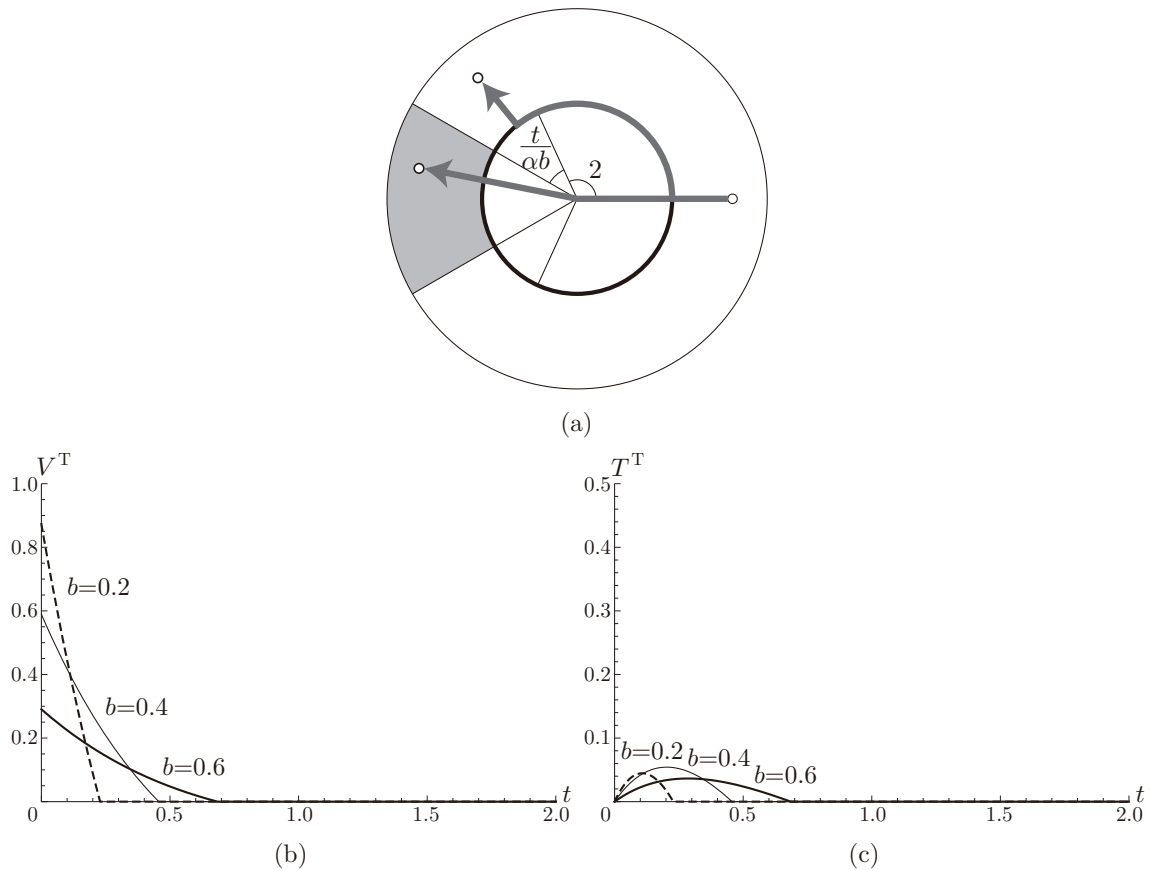


Figure 2: (a) Routes of through traffic; (b) Volume of through traffic passing the toll area; (c) Toll revenue by through traffic

As shown in Figure 2a, if  $2 < \varphi < t/(\alpha b) + 2$ , through traffic makes a detour around the toll area. The travel cost of through traffic making a detour  $C^U$  is given by

$$C^U = \alpha\{r_1 + r_2 + b(\theta_2 - 2)\}. \tag{3.6}$$

Substituting into (2.3) and integrating with respect to  $r_1, \theta_1, r_2,$  and  $\theta_2$  yield the volume of through traffic making a detour

$$\begin{aligned}
 V^U &= \int_0^{2\pi} \int_b^a \int_2^{t/(\alpha b)+2} \int_b^a 2D_0 \exp[-\alpha\beta\{r_1 + r_2 + b(\theta_2 - 2)\}] r_1 r_2 dr_2 d\theta_2 dr_1 d\theta_1 \\
 &= \frac{4\pi D_0}{\alpha^5 \beta^5 b} (e^{\beta t} - 1) \{(\alpha\beta b + 1)e^{\alpha\beta a} - (\alpha\beta a + 1)e^{\alpha\beta b}\}^2 e^{-\beta\{2\alpha(a+b)+t\}}.
 \end{aligned}
 \tag{3.7}$$

The volume of through traffic making a detour  $V^U$  is shown in Figure 3. It can be seen that  $V^U$  increases with  $t$  and then becomes constant. This means that if  $t > t^\dagger$ , all through traffic makes a detour around the toll area. Since this detour traffic passes the boundary of the toll area, the increase in the traffic volume should be considered when designing road pricing.

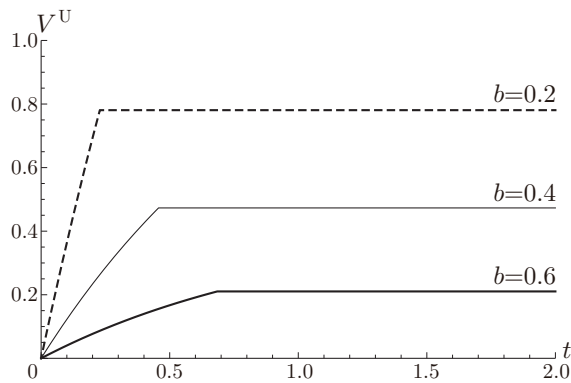


Figure 3: Volume of through traffic making a detour

Let  $(r_1, \theta_1)$  ( $b \leq r_1 \leq a, 0 \leq \theta_1 < 2\pi$ ) and  $(r_2, \theta_2)$  ( $0 \leq r_2 \leq b, 0 \leq \theta_2 < 2\pi$ ) be origin and destination of inward traffic, respectively. Travel routes for trips of  $\theta_1 = 0$  are shown in Figure 4a. The travel cost of inward traffic  $C^I$  is given by

$$C^I = \begin{cases} \alpha(r_1 - r_2 + r_2\theta_2) + t, & 0 \leq \theta_2 < 2, \\ \alpha(r_1 + r_2) + t, & 2 \leq \theta_2 \leq \pi. \end{cases}
 \tag{3.8}$$

Substituting into (2.3) and integrating with respect to  $r_1, \theta_1, r_2,$  and  $\theta_2$  yield the volume of inward traffic

$$\begin{aligned}
 V^I &= \int_0^{2\pi} \int_b^a \int_0^2 \int_0^b 2D_0 \exp[-\beta\{\alpha(r_1 - r_2 + r_2\theta_2) + t\}] r_1 r_2 dr_2 d\theta_2 dr_1 d\theta_1 \\
 &\quad + \int_0^{2\pi} \int_b^a \int_2^\pi \int_0^b 2D_0 \exp[-\beta\{\alpha(r_1 + r_2) + t\}] r_1 r_2 dr_2 d\theta_2 dr_1 d\theta_1 \\
 &= \frac{4\pi D_0}{\alpha^4 \beta^4} \{(\pi - 3 + e^{\alpha\beta b})(e^{\alpha\beta b} - 1) - (\pi - 2)\alpha\beta b\} \\
 &\quad \{(\alpha\beta b + 1)e^{\alpha\beta a} - (\alpha\beta a + 1)e^{\alpha\beta b}\} e^{-\beta\{\alpha(a+2b)+t\}}.
 \end{aligned}
 \tag{3.9}$$

The volume of inward traffic  $V^I$  and the toll revenue by inward traffic  $T^I = tV^I$  are shown in Figures 4b and 4c, respectively. As the toll area becomes larger, both  $V^I$  and  $T^I$  increase. The toll level that maximizes the toll revenue by inward traffic is

$$t^I = \frac{1}{\beta},
 \tag{3.10}$$

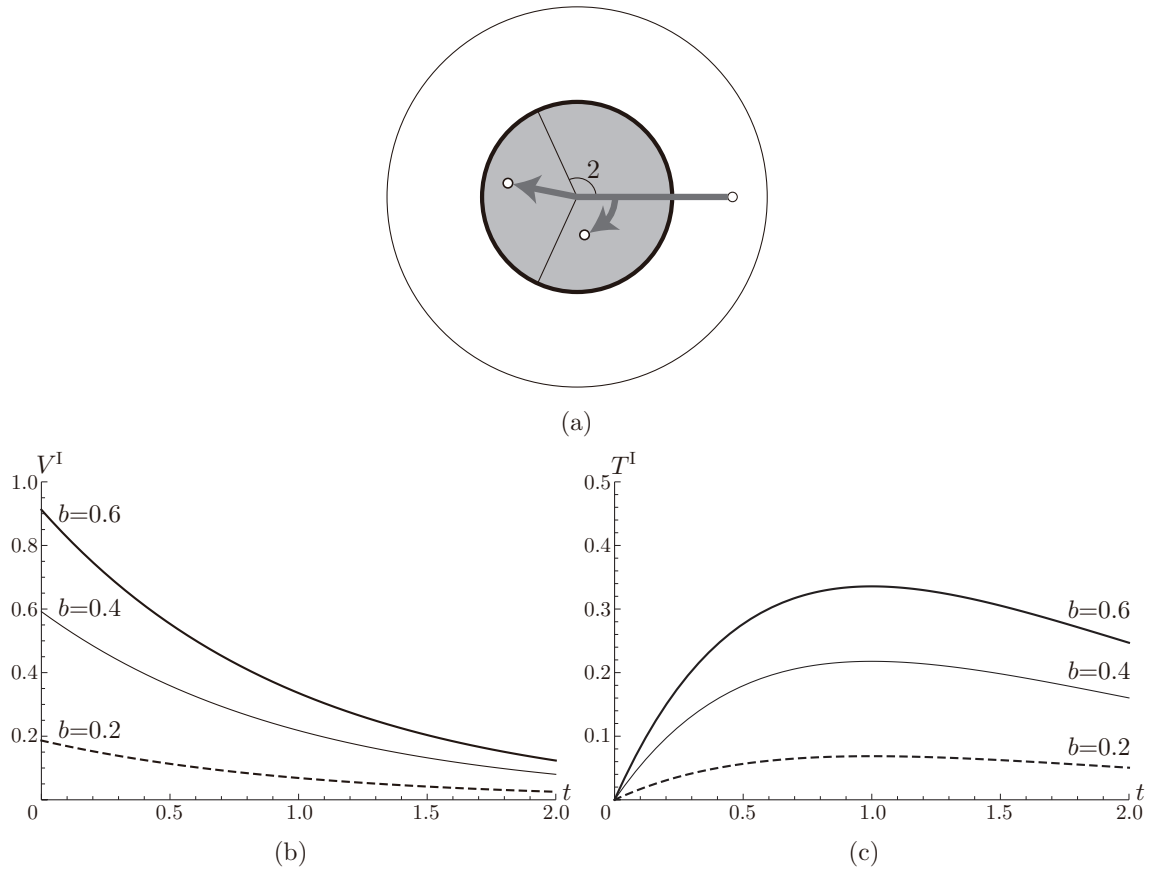


Figure 4: (a) Routes of inward traffic; (b) Volume of inward traffic; (c) Toll revenue by inward traffic

which is independent of the size of the toll area.

The volume of outward traffic  $V^O$  is the same as that of inward traffic, i.e.,

$$V^O = V^I. \tag{3.11}$$

Let  $(r_1, \theta_1)$  ( $0 \leq r_1 \leq b, 0 \leq \theta_1 < 2\pi$ ) and  $(r_2, \theta_2)$  ( $0 \leq r_2 \leq b, 0 \leq \theta_2 < 2\pi$ ) be origin and destination of city traffic, respectively. Travel routes for trips of  $\theta_1 = 0$  are shown in Figure 5a. The travel cost of city traffic  $C^C$  is given by

$$C^C = \begin{cases} \alpha(r_1 - r_2 + r_2\theta_2) + t, & r_1 \geq r_2, 0 \leq \theta_2 < 2, \\ \alpha(r_2 - r_1 + r_1\theta_2) + t, & r_1 < r_2, 0 \leq \theta_2 < 2, \\ \alpha(r_1 + r_2) + t, & 2 \leq \theta_2 \leq \pi. \end{cases} \tag{3.12}$$

Substituting into (2.3) and integrating with respect to  $r_1, \theta_1, r_2,$  and  $\theta_2$  yield the volume of

city traffic

$$\begin{aligned}
 V^C &= \int_0^{2\pi} \int_0^b \int_0^2 \int_0^{r_1} 2D_0 \exp[-\beta\{\alpha(r_1 - r_2 + r_2\theta_2) + t\}] r_1 r_2 \, dr_2 d\theta_2 dr_1 d\theta_1 \\
 &+ \int_0^{2\pi} \int_0^b \int_0^2 \int_{r_1}^b 2D_0 \exp[-\beta\{\alpha(r_2 - r_1 + r_1\theta_2) + t\}] r_1 r_2 \, dr_2 d\theta_2 dr_1 d\theta_1 \\
 &+ \int_0^{2\pi} \int_0^b \int_2^\pi \int_0^b 2D_0 \exp[-\beta\{\alpha(r_1 + r_2) + t\}] r_1 r_2 \, dr_2 d\theta_2 dr_1 d\theta_1 \\
 &= \frac{2\pi D_0}{\alpha^4 \beta^4} [2\alpha^2 \beta^2 b^2 (\pi - 2 + e^{2\alpha\beta b}) + (e^{\alpha\beta b} - 1) \{5 - 2\pi + (2\pi - 11)e^{\alpha\beta b}\} \\
 &\quad - 2\alpha\beta b \{5 - 2\pi + 2(\pi - 4)e^{\alpha\beta b}\}] e^{-\beta(2\alpha b + t)}. \tag{3.13}
 \end{aligned}$$

The volume of city traffic  $V^C$  and the toll revenue by city traffic  $T^C = tV^C$  are shown in Figures 5b and 5c, respectively. If the toll area is small, the toll level has little impact on  $V^C$  and  $T^C$  because  $V^C$  is small even at  $t = 0$ . As the toll area becomes larger, both  $V^C$  and  $T^C$  increase. The toll level that maximizes the toll revenue by city traffic is

$$t^C = \frac{1}{\beta}. \tag{3.14}$$

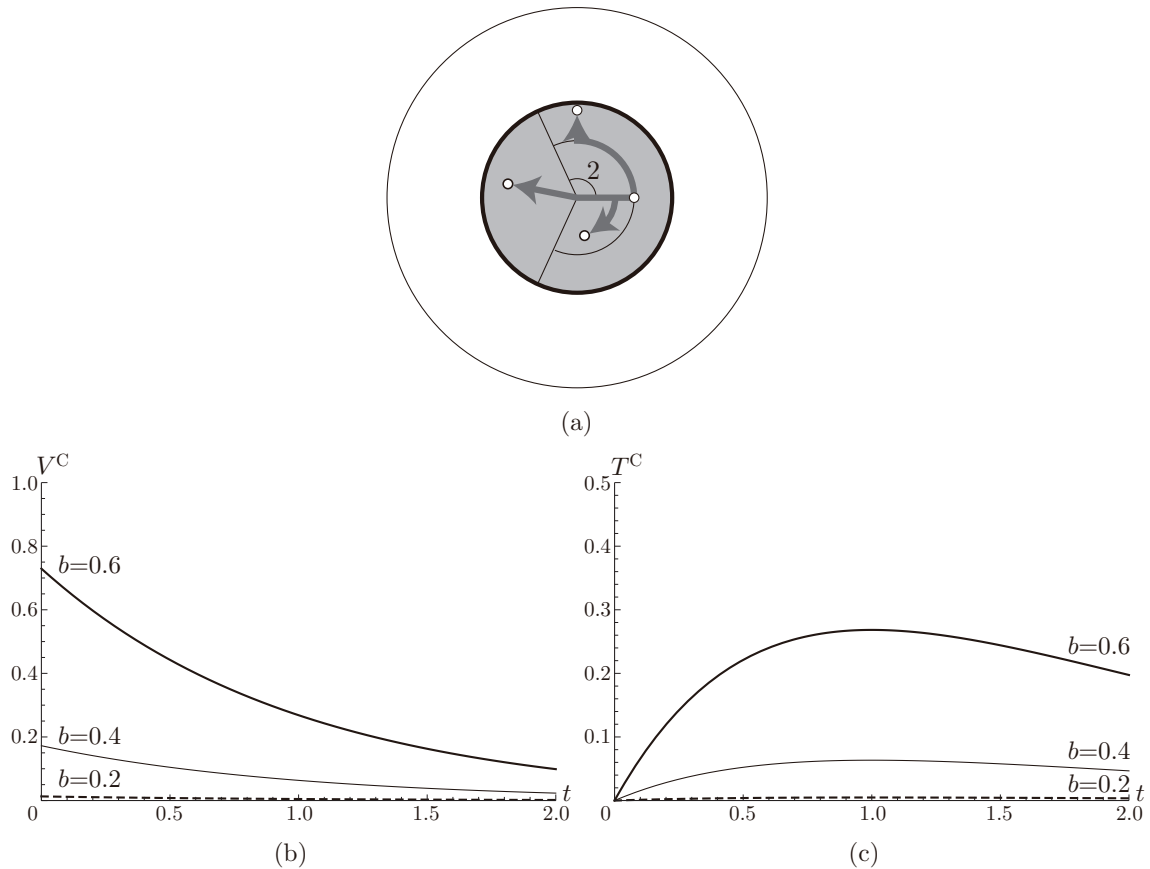


Figure 5: (a) Routes of city traffic; (b) Volume of city traffic; (c) Toll revenue by city traffic

#### 4. Cordon and Area Pricing

Cordon pricing charges a toll to through and inward traffic, whereas area pricing charges a toll to all traffic including outward and city traffic. In this section, we compare the cordon and area pricing in terms of the traffic volume in the toll area and toll revenue.

The traffic volume for cordon pricing is given by the sum of  $V^T$ ,  $V^I$ ,  $V^O$ , and  $V^C$ , where  $t = 0$  for  $V^O$  and  $V^C$ . The traffic volume and toll revenue are shown in Figures 6a and 6b, respectively. Note that both curves have kinks at  $t = t^\dagger$  where the volume of through traffic becomes zero. Note also that the toll revenue has a maximum at  $t^* = 1/\beta = 1$ . These figures provide planners with alternatives for the size of the toll area and toll level. To completely eliminate through traffic from the toll area, the toll level should be set at  $t = t^\dagger$ . To reduce the total traffic volume in the toll area, a higher toll may be required because the traffic volume decreases gradually with  $t$ . If generating revenue is the most important, the toll area should be large and the toll level should be set at  $t = t^*$ .

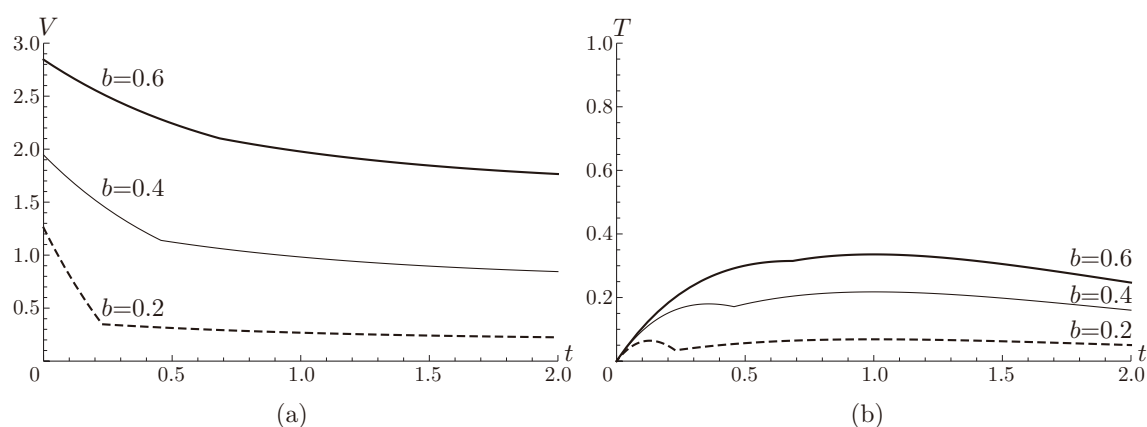


Figure 6: (a) Traffic volume for cordon pricing; (b) Toll revenue for cordon pricing

The traffic volume for area pricing is given by the sum of  $V^T$ ,  $V^I$ ,  $V^O$ , and  $V^C$ . The traffic volume and toll revenue for area pricing are shown in Figures 7a and 7b, respectively. Note that as the toll level increases, the traffic volume for area pricing decreases more rapidly than that for cordon pricing. Note also that the toll revenue for area pricing, which has a maximum at  $t^* = 1/\beta = 1$ , is greater than that for cordon pricing. It follows that area pricing is superior to cordon pricing in both reducing traffic volume in the toll area and generating revenue. This is consistent with the result of case studies using actual road networks and traffic data [11, 12].

#### 5. Conclusions

This paper has presented a continuous network model for determining the size of the toll area and toll level in road pricing. The traffic volume and toll revenue have been obtained for a circular city with a radial-arc network. Cordon and area pricing have then been compared in terms of the traffic volume in the toll area and toll revenue.

The model is useful for road pricing design as follows. First, the analytical expressions for the traffic volume and toll revenue demonstrate the effects of the size of the toll area and toll level. Note that finding these relationships by using discrete network models requires computation of the traffic volume for various combinations of the parameters. These relationships help planners to estimate the size of the toll area and toll level required to achieve a certain level of traffic volume and toll revenue. Second, the model explicitly considers



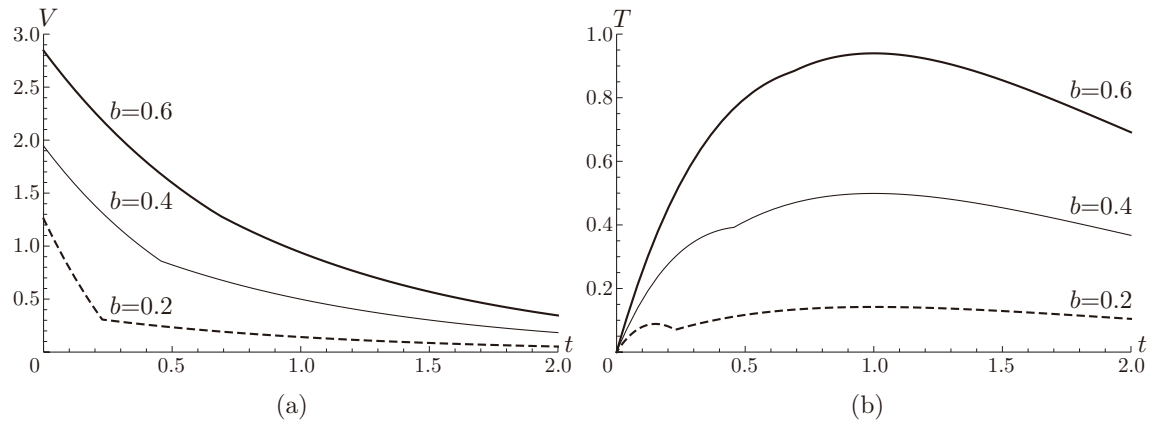


Figure 7: (a) Traffic volume for area pricing; (b) Toll revenue for area pricing

through, inward, outward, and city traffic, thereby allowing us to examine how the traffic composition affects the traffic volume and toll revenue. Finally, comparing cordon and area pricing shows that area pricing is superior to cordon pricing in both reducing traffic volume in the toll area and generating revenue. This result provides a fundamental understanding of road pricing and gives an insight into discrete network models for empirical analysis.

To obtain analytical expressions, the present model has many simplifying assumptions. Future research should extend the model to incorporate non-uniform distribution of origins and destinations, travel demand based on trip-chain, and multi-layered cordon.

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