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Financing, investment, liquidation, and costly reversibility

01208080 首都大学東京 *芝田 隆志 SHIBATA Takashi
01406054 大阪大学 西原 理 NISHIHARA Michi

1. Introduction

We examine the interaction between financing and investment decisions under the condition that debt holders have the option of maximizing the debt-collection amount if a firm is liquidated during financial distress.

2. Model

A firm possesses an investment option. When the option to invest is exercised, the firm receives an instantaneous cash inflow $(1-\tau)qX(t)$ after investment, where $\tau > 0$ represents the tax rate, q represents the investment quantity, and $X(t)$ represents the stochastic price given by the geometric Brownian motion

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}z(t),$$

where $X(0) = x > 0$, $\mu \in (0, r)$, $\sigma > 0$, $z(t)$ denotes a standard Brownian motion, and $r > 0$ is the risk-neutral interest rate.

When the investment option is exercised, the firm incurs an investment cost $I(q) > 0$. We assume $I(0) > 0$, $I'(q) > 0$, $I''(q) > 0$, and $(qI'(I)/I(q))' > 0$.

After investment, the firm has the option to shut down the project and sell the used production facility at a resale price of $(s + k)I(q)$, where $s \in [0, 1]$ and $k \in [0, 1) \cap [0, 1-s]$ represent the "costless reversibility" ratio and "costly reversibility" ratio, respectively. Thus, $(1-s-k)I(q)$ indicates the depletable (disposal) ratio.

Let $g(k) \geq 0$ denote the cost function of costly reversibility. We assume that $g(k)$ satisfies four conditions: $g(0) = 0$, $g'(k) > 0$, $g''(k) > 0$, and $\lim_{k\to 1} g(k) = +\infty$.

In addition, we assume that the firm incurs a shutdown ("liquidation bankruptcy") cost $\alpha((s + k)I(q) - g(k)) \geq 0$, representing some portion of the liquidation value, where $\alpha \in (0, 1)$. Thus, the liquidation value is defined as $(1 - \alpha)((s + k)I(q) - g(k)) \geq 0$.

Now, suppose that the firm stops operating. At the time of liquidation, the optimization problem is formulated as

$$\begin{aligned} L(q) &:= \max_{k\in[0,1-s]} (s + k)I(q) - g(k), \qquad (1) \\ &= (s + k(q))I(q) - g(k(q)) \geq 0, \end{aligned}$$

where the optimal costly reversible ratio, $k(q)$, is obtained by

$$k(q) = \qquad (2)$$
$$\begin{cases} 0, & \text{if } I(q) \in (0, g'(0)), \\ g'^{-1}(I(q)), & \text{if } I(q) \in [g'(0), g'(1-s)], \\ 1-s, & \text{if } I(q) \in (g'(1-s), +\infty). \end{cases}$$

Assume the case of $c \in [0, \theta_1(q)]$ where $c \geq 0$ denotes the coupon payment and $\theta_1(q)$ is

$$\theta_1(q) := r(1-\alpha)L(q) \geq 0. \qquad (3)$$

Then, debt is risk-free. After investment, the value of the risk-free debt, $D_1(X(t), q, c)$, is equal to

$$D_1(X(t), q, c) = \frac{c}{r} \geq 0. \qquad (4)$$

The value of the equity, $E_1(X(t), q, c)$, is

$$\begin{aligned} E_1(X(t), q, c) &= vqX(t) - (1-\tau)\frac{c}{r} \qquad (5) \\ &+ \Big((1-\alpha)L(q) - vqx_1^{\mathrm{s}}(q, c) - \tau\frac{c}{r}\Big)\Big(\frac{X(t)}{x_1^{\mathrm{s}}(q, c)}\Big)^{\gamma}, \end{aligned}$$

where $v := (1-\tau)/(r-\mu) > 0$, $\gamma := 1/2 + \mu/\sigma^2 - ((\mu/\sigma^2) - 1/2)^2 + 2r/\sigma^2)^{1/2} < 0$, and the optimal shutdown trigger $x_1^{\mathrm{s}}(q,c)$ is

$$x_1^{\mathrm{s}}(q,c) = \frac{\varepsilon}{q}((1-\alpha)L(q) - \tau\frac{c}{r}) \geq 0, \qquad (6)$$

where $\varepsilon := \gamma/((\gamma-1)v) > 0$.

Assume the case of $c \in (\theta_1(q), +\infty)$ (i.e., debt is risky). After investment, the equity value $E_2(X(t), q, c)$ is

$$E_2(X(t),q,c) = vqX(t) - (1-\tau)\frac{c}{r} + \Big((1-\tau)\frac{c}{r} - vqx^{\mathrm{d}}(q,c)\Big)\Big(\frac{X(t)}{x^{\mathrm{d}}(q,c)}\Big)^{\gamma},$$

where the optimal default trigger $x^{\mathrm{d}}(q,c)$ is

$$x^{\mathrm{d}}(q,c) = \frac{\varepsilon}{q}\frac{1-\tau}{r}c > 0. \qquad (7)$$

The market value of a risky debt after investment, $D_2(X(t), q, c)$, is given by

$$D_2(X(t),q,c) = \frac{c}{r} + \Big(W(x^{\mathrm{d}}(q,c),q) - \frac{c}{r}\Big)\Big(\frac{X(t)}{x^{\mathrm{d}}(q,c)}\Big)^{\gamma}. \qquad (8)$$

Here, $W(x^{\mathrm{d}}(q,c), q)$ is

$$W(x^{\mathrm{d}}(q,c),q)/(1-\alpha) = \begin{cases} L(q), & c \in (\theta_1(q), \theta_2(q)], \\ vqx^{\mathrm{d}}(q,c) + (L(q) - vqx_2^{\mathrm{s}}(q))\Big(\frac{x^{\mathrm{d}}(q,c)}{x_2^{\mathrm{s}}(q)}\Big)^{\gamma}, & c \in (\theta_2(q), +\infty), \end{cases}$$

where $x_2^{\mathrm{s}}(q) := \mathrm{argmax}_y\, vqx^{\mathrm{d}}(q,k) + (L(q) - vqy)(x^{\mathrm{d}}(q,c)/y)^{\gamma}$ and $\theta_2(q)$ are obtained as

$$x_2^{\mathrm{s}}(q) = \frac{\varepsilon}{q}L(q) \geq 0 \qquad (9)$$

and

$$\theta_2(q) := r(1-\tau)^{-1}L(q)(\geq \theta_1(q) \geq 0), \qquad (10)$$

respectively.

$(1-\alpha)L(q)$	c/r	0.7228
	x^{i}	0.3416
	q	0.7629
$(1-\alpha)L_N(q_N)$	c_N/r	−0.8206
	x_N^{i}	−0.9491
	q_N	0.1566

Table 1: Correlation with liquidation value

3. Firm's optimization problem

The firm's problem is

$$\max_{x^{\mathrm{i}}>0, q\geq 0, c\geq 0} J_m(x^{\mathrm{i}}, q, c), \qquad (11)$$

where $\beta := 1/2 + \mu/\sigma^2 + ((\mu/\sigma^2) - 1/2)^2 + 2r/\sigma^2)^{1/2} > 1$ and

$$J_m(x^{\mathrm{i}}, q, c) := {x^{\mathrm{i}}}^{-\beta}\{V_m(x^{\mathrm{i}}, q, c) - I(q)\}, \qquad (12)$$

where $V_m := E_m + D_m$ ($m \in \{1, 2\}$).

4. Results

This section numerically considers the important implications of our solution. Here, we assume $I(q) > 0$ and $g(k) \geq 0$ as

$$I(q) = F + q^2 > 0, \qquad (13)$$

$$g(k) = a\frac{k}{1-k} \geq 0, \qquad (14)$$

where $F > 0$ and $a > 0$.

Table 1 shows the correlation. Suppose the the firm has the option of costly reversibility ($k \geq 0$). The higher the liquidation value, the larger the debt issuance, the more delayed the investment timing, and the larger the investment quantity. On the other hand, suppose that the firm does not have the option. The higher the liquidation value, the smaller the debt issuance, the more accelerated the investment timing, and the smaller the investment quantity. Empirical results fit better with our theoretical ones than previous ones. We will show other interesting results at the presentation.

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